B.Sc. Semester-IV Examination, 2022-23

MATHEMATICS [Programme]

Course ID : 42118 **Course Code : SP/MTH/401/C-1D**

Course Title : Differential Equations and Vector Calculus

Time : 2 Hours Full Marks : 40

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer any **five** from the following questions: 2×5=10

a) Illustrate by an example that a continuous function may not satisfy a Lipschitz condition on a rectangle.

b) Show that e^{2x} and e^{3x} are linearly independent solutions of the differential equation

$$\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$$

c) Find the singular solution, if any of $y = 2px+p^2$ where $p=\frac{dy}{dx}$.

[Turn Over]

d) Convert the second order differential equation

$$2\frac{d^2y}{dx^2}-5\frac{dy}{dx}+4y=0$$

as a system of first order linear differential equation.

e) Solve: $xdx + ydy = k(xdy - ydx)$.

f) A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$, where t is the time. Find the magnitudes of the velocity and acceleration at $t = 0$.

g) Show that $\nabla\varphi$ is a vector perpendicular to the surface $\varphi(x,y,z)=c$, where c is a constant.

2. Answer any **four** from the following questions: 5×4=20

a) Solve by the method of variation of parameter, the equation $\frac{d^2y}{dx^2}+a^2y=\tan ax$.

b) Solve the differential equation

$$\left(3D^2+2D-8\right)y=5\cos x.$$

c) Use operator method to solve the linear system

$$\frac{dx}{dy}+4x+3y=t;\ \frac{dy}{dt}+2x+5y=e^t.$$

d) Find the general solution of the linear autonomous system

$$\frac{dx}{dt} = 3x + y; \quad \frac{dy}{dt} = 3x - y.$$

Determine the nature of the critical point of the system and also comment on the stability.

e) Suppose a force field is given by

$F = (2x - y + z)\hat{i} + (x + y - z^2)\hat{j} + (3x - 2y + 4z)\hat{k}$.

Find the work done in moving a particle once around a circle C in the xy-plane with its centre at the origin and a radius of 3.

f) Verify Green's theorem in the plane for $\oint (2x - y^3)\,dx - xy\,dy$ on the boundary of the region enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$.

3. Answer any **one** of the following questions:

10×1=10

a) i) Solve and find the singular solution of the differential equation $(px - y)(x - py) = 2p$, where $p = \frac{dy}{dx}$.

ii) Prove $\nabla \times (\nabla \times A) = -\nabla^2 A + \nabla(\nabla . A)$.

6+4=10

b) i) Prove that $\frac{1}{(x+y+1)^4}$ is an integrating factor of $(2xy - y^2 - y)dx + (2xy - x^2 - x)dy = 0$ and hence solve it.

ii) Find the directional derivative of the divergence of $\vec{f}(x, y, z) = xy\hat{i} + xy^2\hat{j} + z^2\hat{k}$ at the point (2, 1, 2) in the direction of outer normal to the sphere $x^2 + y^2 + z^2 = 9$.

6+4=10

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